



Coles, A. T. (2015). On enactivism and language: Towards a methodology for studying talk in mathematics classrooms. *ZDM*, 47(2), 235-246. <https://doi.org/10.1007/s11858-014-0630-y>

Peer reviewed version

Link to published version (if available):
[10.1007/s11858-014-0630-y](https://doi.org/10.1007/s11858-014-0630-y)

[Link to publication record in Explore Bristol Research](#)
PDF-document

The final publication is available at Springer via <http://dx.doi.org/10.1007/s11858-014-0630-y>

University of Bristol - Explore Bristol Research

General rights

This document is made available in accordance with publisher policies. Please cite only the published version using the reference above. Full terms of use are available:
<http://www.bristol.ac.uk/red/research-policy/pure/user-guides/ebr-terms/>

On enactivism and language: towards a methodology for studying talk in mathematics classrooms

Alf Coles

Abstract This article is an early step in the development of a methodological approach to the study of language deriving from an enactivist theoretical stance. Language is seen as a co-ordination of co-ordinations of action. Meaning and intention cannot easily be interpreted from the actions and words of others, instead, careful attention can be placed in not going beyond what is observable within the text itself, for example by focusing on patterns in word use. Conversations are highly ritualised affairs and from an enactivist perspective these rituals can be read in terms of pattern. The notion of the 'structural coupling' of systems, which will inevitably have taken place in a classroom, means that the history and context of communication needs to be taken into account. The methodological perspective put forward in this article is exemplified with an analysis of two classroom incidents (involving different teachers) in which almost identical words are used by the teachers, but markedly different things happen next. The analysis reveals a complexity within the classroom that, although available to direct observation, only became apparent using an approach to studying language that took account of the context and history of communication in a recursive process of data collection and analysis.

1 Introduction

This article sets out an initial step towards the development of a methodological approach to the study of language, derived from enactivism, and then investigates its use. The ideas that follow are the result of grappling with ways of making sense of data that I collected as part of a project funded by the UK's Economic and Social Research Council (ESRC). At the start of the ESRC grant, I was working full time as head of mathematics in a secondary school in the UK. The purpose of the project was to investigate both the use of video as a tool for professional development, and video recordings of mathematics lessons at the school. I encouraged all members of the department to take video recordings of their teaching and to share excerpts at staff department meetings. For the ESRC project, video recordings were taken over two years in the classrooms of two teachers. I had wanted to compare and contrast more than one teacher in order to look for similarities and differences. I chose the two teachers partly because they volunteered but also I had in mind that I wanted to observe one teacher who was experienced in the department and one who had just joined, to explore what features there might be of common practice. I was interested in how particular classroom environments get established from the beginning of the academic year. As a department, we had agreed a shared focus of working together on 'ways of managing student talk'. When looking at the video recordings of lessons I was interested, therefore, in times of whole class discussion and, particularly, when there seemed to be some disruption to a pattern of talk – when something happened that required the teacher to engage in overt 'management' of the discussion, perhaps adapting or changing what they were planning. I came to need a way of approaching these recordings of classroom talk which might help me say something about how, as teachers, we can support mathematical thinking and develop classroom environments in which students become creative and independent in their work. There are ethical concerns around taking a dual role, as I did, of researcher and head of department. My own purposes within both roles were to some extent aligned in that as a researcher I was interested in how to make discussion of video meaningful for participants, and as a head of department I was concerned with the professional development of staff. The tensions came around my own watching of video recordings. As head of department I could be asked to comment to senior leaders on the teaching of mathematics staff. I was mindful to move away from any deficit view of comparing lessons or teacher actions and to get to a place (as

researcher and head of department) of more neutrally observing what took place, rather than what did not.

In what follows, Section 2 sets out my enactivist stance, with a particular focus on language. Section 3 draws out the methodological implications for the study of language in mathematics classrooms, making links and pointing out differences to similar approaches. In Section 4, the value of the approach is investigated through an analysis of classroom data, leading to conclusions in Section 5.

2. Enactivism and languaging

Enactivism, as a theoretical framework, draws from the traditions of phenomenology (Merleau Ponty 2002), cybernetics (Bateson 2002) and a radical view of biology (Maturana & Varela 1987). The enactive view of language is in the tradition of Wittgenstein (2009) in viewing words as tools. Learning our first language is an entry into a world that is (initially at least) not of our own choosing. We do not so much learn the meanings of words as we ‘catch on to them ... much as one imitates a gesture’ (Merleau Ponty 2002, p.469). The language of a community fits the activities of that community. Language learning is entering into a community with a particular way of living, or ‘form of life’ (Wittgenstein 2009, p.15). We enter a community as we enter into the use of its language and engage in typical actions. Words gain their generality from the typicality of the world and our responses.

Maturana (1988) introduces the notion of a ‘linguistic domain’ to describe how different groups of people establish manners of co-ordinating the negotiation of their actions. Maturana reasoned that language arises only when two or more organisms are *already* in relationship. It is only when organisms have become part of the medium for the living of each other, that common ‘objects’ arise that can be denoted by words. Hence, it is only when the very structure of organisms become coupled to each other, over the history of a species and a particular lifetime, that there is the possibility of language. Language must first support the on-going relationship between organisms, before any denotative function can arise. Systems co-evolve when involved in recursive interactions. System and medium change congruently, ‘as a spontaneous consequence’ of interaction, the structural congruence that emerges is labelled ‘structural coupling’ (Maturana & Verden-Zoller 2008, p.26-7).

Structural coupling entails co-ordination of action. Sharing a language is one aspect of the structural coupling that occurs when humans live in relationship. Through language, we are able to establish consensus and co-ordination *about* the domain of our actions. In other words, language is a second-order domain, or a co-ordination of co-ordination of action. Meaning arises through, and is interpreted as, the co-ordination of action:

[o]bjects arise in language ... [they] constitute a field of shared co-ordinations of doings ... which is felt by those who live it as a domain of shared entities. (Maturana & Verden-Zoller 2008, p.35)

Objects are not in the world waiting to be named, they are created (for an individual or group) through activity and language. An example of a short one-to-one teaching sequence may help illustrate the layered nature of action, meaning and language. I was the teacher, working with a student (my daughter). I took notes shortly afterwards, from which the indented text is reconstructed.

1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900

Figure 1: A tens chart

Looking at the chart in Figure 1, the teacher says: “I am going to tap on two numbers. What do I multiply by to get from first to second?”. The teacher taps with a small stick on 1, then 10, the student responds: “times 10”. The teacher points to 3, then 30; student: “10”. The teacher taps on 40, then 400; student: “times 10”. After a few more repetitions, the teacher points to 10 followed by 1. The student says “10”; teacher: “what do you multiply 10 by to get 1?”. After no response, the teacher says: “this is times by a tenth”; student: “a tenth ... a tenth ... a tenth” (the interaction continued).

As the teacher points, there is shared activity, both teacher and student follow the pointer with their eyes. A pattern in the pointing (1,10), (2,20), (40,400) is repeated and there is the potential for the creation of meaning. Without using language to describe it, a shared meaning could be verified, for example, by getting the student to ‘tap’ a ‘jump’ that was the ‘same’. With few or no words, there is the potential for the co-ordination of action that indicates a shared entity is emerging (that an observer might describe as ‘move down one row’). With the introduction of language in relation to this shared entity (‘multiply by 10’ and, later, ‘multiply by a tenth’) new possibilities arise. The words provide the potential to co-ordinate the meanings that have arisen from interaction with the chart. For example, connections between movements ‘down’ and ‘up’ can be made; the movements can be iterated (how do you describe moving down two rows?); links can be made to other images or senses of ‘multiply by 10’, or to writing down what is being done. In all these examples, language co-ordinates meaning in the sense of relating meanings to other meanings, language is a second-order co-ordination. The use of language is part of the on-going relationship between teacher and student.

It is through shared activity that we create objects in language, with those around us that may or may not be named. These objects appear to be shared entities, but it is an implication of Maturana’s view of our biology that each system necessarily experiences its medium in a unique way. We live in a ‘multi-verse’ (not universe). We establish shared coherences through structural coupling; in other words, we co-ordinate our actions in relation to specific parts of our environment (e.g., tapping numbers on a tens chart), which we can name, refer to and reflect on through language. The naming of shared entities captures distinctions made in the world (e.g., noticing the similarity in the movement (2,20) and (20,200)). As Steinbring (2015) notes, successful communication does not imply participants share ‘personal understanding’, rather through joint activity we can come to share a common conceptual background. Or to put this another way, the teacher has no access to what the student perceives or notices, all we can hope to do, in learning, teaching or researching, is refine and work on how we make distinctions, through joint activity and reflecting with others. Maturana frequently refers to ‘languaging’ rather than ‘language’ (see Reid and Mgombelo, 2015); the verb highlights the connection between communication and our on-going co-ordination, co-evolution and living in our world.

2.1 Links to other approaches to language

An approach to social science close to the enactivist view is ethnography. The idea of altering our distinctions, or ways of seeing, is close to the ethnographic notion of developing ‘sensitizing concepts’ (Blumer 1954, p.7) which rather than provide ‘prescriptions of what to see ... suggest directions along which to look’ (ibid). Ethnographic analysis tries to stay alert to what is left out, i.e., staying open to the possibility of altering one’s categorisations. Enactivist research is similarly concerned with developing ways of seeing data, developing ever richer and finer distinctions, as I hope the reader may experience by the end of this article.

Ethnography gave rise to ‘conversation analysis’ (Sacks 1995) when it was discovered that conversations are highly ritualised affairs (Schelgoff 1989) and that it is possible, for example, to read meanings in the way turns of a dialogue are accepted or rebutted. Within conversation analysis, therefore, searching for patterns in language use is a key to making sense of dialogue.

Conversation analysis is cited as background to two other approaches to studying language, namely linguistic ethnography (Rampton 2006) and discursive psychology (Edwards 1997). Linguistic ethnography (LE) hosts a broad group of researchers who share two central assumptions. The first is that ‘the context for communication should be investigated rather than assumed’ (Rampton 2007a, p.585), which implies the researcher may need to bring a ‘reflexive sensitivity to the processes involved in the production of linguistic claims and to the potential importance of what gets left out’ (2007a, p.596). To be sensitive, as a researcher, to the process by which our own linguistic claims are produced, in part means paying attention to the ‘taken as read’ particularly when, as in my case, I was part of the context of video recordings and staff meetings. One mechanism for applying such reflexive sensitivity is to consider, in a systematic manner, ‘what gets left out’ in making any statement (an idea that is expanded on in Section 3.4). This assumption links to the enactivist notion of the structural coupling of systems that are in recurrent interaction, which implies the context of communication is not a passive background to talk but rather an active and changing component.

The second assumption is that meaning ‘is far more than just the ‘expression of ideas’, and biography, identifications, stance and nuance are extensively signalled in the linguistic and textual fine-grain’ (Rampton 2007a, p.585). It is not necessary to bring concepts such as ‘identification’ to an analysis of talk (other concepts here might be ‘power’, or ‘community’). It is possible to begin from the fine-grained detail and assume that if more abstract concepts are relevant to the data, they will be ‘extensively signalled’ in this fine-grain. The sense of meaning as, ‘more than just the ‘expression of ideas’’ is consistent with enactivism’s rejection of communication as the denotation of objects or transmission of information (Maturana & Varela 1987, p.196). To move beyond seeing interactions as the ‘expression of ideas’, researchers within LE have a variety of techniques. The strand of micro-analysis, set out in more detail in Section 3.4, is the one that most directly addresses the need to dwell in the detail of communication.

There are other perspectives on language in use within a mathematics education research context, which share one or both of the assumptions above. For example, Herbel-Eisenmann and Wagner (2010)’s ‘Appraisal Linguistics’ makes use of computer recognition of common word usage in secondary mathematics classrooms to denormalize ‘pervasive discourse patterns’ (p.44) and allow critical reflection on alternatives. Such a process fits well with the first assumption, above, and the need to generate reflexive sensitivity. Appraisal Linguistics is an approach within Systemic Functional Linguistics (SFL). For SFL, language (as a ‘system’) is seen as, ‘a resource that you can draw on in reading and speaking and listening – and a resource that you use for learning with’

(Halliday 1999, p.7). We act on language in the form of ‘text’, which ‘refers to all the instances of language that you listen to and read. And that you produce yourself in reading and in writing.’ (ibid).

Halliday contrasts ‘text’, the instances of language use, with the non-verbal ‘context’ which is relative to a particular culture and to a particular instance of use within that culture. In a critique of SFL, de Freitas (2010) questions Halliday’s distinction between text and context, suggesting it creates ‘an unexamined ontological divide’ (p.133) that is rejected by postmodern perspectives which:

have gone further in problematizing the distinction, revealing how there is no getting outside of text ... there is no pre-textual context, no subject outside of text, and no pure text disengaged from other texts (p.134).

Enactivism endorses such a postmodern perspective, not so much in the sense that everything is interpreted as text but because from an enactive stance system/medium and text/context arise together and cannot be studied in isolation. It is through acting in the world that value and meaning emerge (Thompson & Stapleton 2009); if we study just the actions of a system, or just the response of its medium, something essential is lost. Reid (1996) comments that, for enactivist methodology, ‘*there is no data, only interpretations and interpretations of interpretations*’ (1996, p.206, italics in original), disrupting too sharp a distinction between what we think a shared entity ‘means’ and the entity itself, an insight which can be translated into a disruption of the distinction between text (data) and context (through which we try to make sense of the data).

An enactive approach to language aligns closely with a notion in discursive psychology of ‘unmotivated looking’ (Edwards 1997, p.89). The analytic principle here, Edwards continues, is: to avoid ‘reading into’ data a set of ready-made analytic categories: to avoid being disappointed with the data available, and to allow ‘interesting’ to be a matter of whatever analysis can produce (1997, p.89).

The point is not that I can ever see the data ‘as it really is’ (an idea that makes no sense from an enactive point of view) but that there is a difference if I approach data looking for something (e.g., a fit with a categorisation) or if I do not. Herbel-Eisenmann and Wagner’s (2010) use of computer software to pick out patterns in text is one way of taking the motivation out of looking, once the overall framework has been decided. The sense of unmotivated looking is important because there is a danger for any researcher that we will only see in terms of our existing categories. In general, as humans, we only notice what is already in our minds. Maheux and Proulx (2015) flag the importance of attending to our data *as* observations. To put this another way, we cannot stand apart from the processes we study, therefore, *how* we use ourselves, as an observer, matters and needs to be made explicit (Coles 2009). It makes a difference to what I see from the back of a classroom if, say, my intention is to record as much detail as possible of what the teacher says, or if I have a list of characteristics I am looking out for.

There is one commitment that sets the enactive view apart from SFL, discursive psychology and linguistic ethnography and that is an interest in learning. Discursive psychology and linguistic ethnography are clear in wanting to avoid commitment to a particular view of cognition, hence they exclude discussion of learning. Enactivism, by contrast, is quite centrally a theory of cognition. From an enactive perspective, learning and knowing are closely linked to action and therefore can be read in the actions of others. Maturana and Varela (1987), state, ‘cognition is effective action, an action that will enable a living being to continue its existence in a definite environment’ (1987, p. 29). So an action is effective if it allows me to continue operating in a specific context. Knowing,

which requires cognition, is therefore also equated with effective action. Action and perception cannot be dissociated, so learning is associated with change and the arising of new actions/perceptions in a given context. A new description of a situation or a new way of seeing would count, on the enactive view, as learning. Enactivist researchers seek out multiple views of data (Reid 1996) precisely to expand ways of seeing.

A final feature of the enactive view of language that is significant for research purposes is a commitment to the importance of meta-communication. Bateson (2000, p.247) drew attention to the way that all messages carry metamessages about how the message is to be read and about the relationship implied between the participants. Such metamessages are carried by facial expressions, body position, tone of voice and no doubt are generally unconscious. We 'frame' our interactions through these metamessages in such a way that is vital for the co-ordination of meaning. On occasion we also communicate about our communications and make some of this framing explicit. Such communication about communications, Bateson calls metacommunication. Of course our metacommunications and communications can contradict (placing the hearer in a bind) however, we can interpret some of the context of any communication through the metamessages and metacommunications used by participants.

3. Methodological implications for the analysis of data

The point of setting out the enactive view of language, above, is to be able to consider the methodological implications. If language is seen as the co-ordination of co-ordination of action, or the co-ordination of meaning, what does this imply for the analysis of talk in a mathematics classroom?

There is a tension, from an enactivist perspective, between wanting to focus on the detail of communication and wanting to take account of context. There is a commitment not to go beyond what is observable, that what counts should be signalled in the fine grain. At the same time, there is a commitment to the view that language arises out of the structural coupling of organisms. In other words, having some awareness of, or access to, the history of interaction or the context of communication is vital in order to be able to make sense of the fine grain of talk.

In the next pages, I put forward five mechanisms that allow an approach to language and learning, consistent with an enactive view. These mechanisms have been developed through use (see Brown & Coles 2011; Coles 2013a) and do not cover every aspect of analysis, for example, they do not touch on the issue of transcription, which can be highly significant (Ochs 1979). The list is offered in the spirit of initial ideas:

1. Recursive inquiry
2. The systematic search for pattern
3. Equifinality
4. Micro-analysis
5. Meta-communication

Each mechanism will be elaborated, briefly, and exemplified with classroom examples in section 4. Each mechanism on its own is not unique to enactivism, but their combination and the overall interest in learning only makes sense from an enactivist perspective.

3.1 Recursive inquiry

One way of addressing the tension between a focus on detail and an awareness of context is to have some structure for a recursive process of data collection and data analysis over time, so that insights inform future work during the lifetime of any project. The choice of the word ‘recursive’ is to indicate that no sharp division is being drawn between data collection and analysis (nor between text and context). Recursion suggests a repeated interaction, with results from one iteration feeding into the next. For example, in working with a group of teachers over time and taking video recordings of lessons, two lesson transcripts might form the basis of a meeting with the teachers, where data will be analysed and that analysis form part of the data collection. Where possible roles will be joint, e.g., teachers might also have responsibilities as researchers.

The notion of member checking (e.g., Herbel-Eisenmann & Wagner 2010, p.50) is a common research practice that also results in data or analysis being fed back to research participants and is an aspect of the ethical dimension of any project. The ethics of enactivist research require (beyond the standard consents and ethical board requirements) an on-going attention to the human relationship being developed (Varela 1999). It is not necessarily the case that this means it is the ethical thing to share data with participants.

For example, within the department the research reported on in this article took place, our express aim was to engage students in a focus on mathematics and mathematical relationships. We sought to exclude from the classroom, where possible, anything that distracted from this aim. In particular, we would not want to provoke unnecessary awareness about the manner of learning mathematics (for example, that it was unusual). Sharing video transcript data with students (particularly if transcripts of two different classrooms were shared) would be likely to provoke awareness of and interest in the similarities and differences between classrooms. Some students might prefer the sound of the other classroom. Focus would be taken away from mathematics in a manner that could have lasting effects. In contrast, a key focus of my relationship with staff was on developing awareness of differences between classrooms and the role of the teacher, hence in this project, the ethical decision was to share data with teachers but not students.

3.2 The systematic search for pattern

In the search for pattern there is a need to split or segment data in a systematic manner. Audio data from a classroom can be separated in any number of ways depending on the focus or interest of the research(er). To make the search for pattern systematic, decisions need to be made in advance. Having identified sections of the data to be the focus for analysis, the enactive view of language implies then looking for observable similarities or differences. This stage of analysis may involve some quantitative methods, for example, word or phrase counts to identify commonalities (as done by others, e.g., Herbel-Eisenmann et al. 2010).

The search for pattern is not about wanting to ensure that everyone looking at the same data would make the same distinction. From an enactive perspective, what matters is that once a pattern is noticed, it must be observable by others (not that others would always notice it in the first place). There is no requirement for the ‘vast number’ of cases required for the emergence of reasoning (Khan, Francis and Davis 2015). A pattern could be comprised of as few as two instances.

To give an example, in one video (taken as part of the ESRC project), I was struck by the use of the word ‘conjecture’ by students. My own history of structural coupling within mathematics classrooms of course sensitised me to the use of this word – another researcher looking at the same video may not have noticed it. This aspect of the search for pattern could not be called unmotivated. However, having become interested in the word, I looked back through previous video recordings to

see if there were further instances of its use (a search that was unmotivated). I discovered that in the first video of the academic year with this class, the teacher used the word ‘conjecture’ thirty-six times in a 20 minutes whole class discussion. I then tracked the use of the word through all the video recordings I had, to see how the use changed (see Coles 2013b).

The search for pattern, where possible, includes multiple researchers, in seeking multiple views of data, as mentioned above (Reid 1996, p.206). Different researchers each interrogate the same data, looking for different things. For example, in a transcript of a teacher meeting, one researcher might look at all sections where teachers talk about their students, another might look at all the sections where judgments are made. Sections of transcript that come up as significant from multiple perspectives are privileged, and might be interrogated further. Patterns are then sought within and across sections of talk, in the search for ‘the pattern that connects’ (Bateson 2002, p.10). An example of a connecting pattern is offered in Section 3.5.

3.3 Principle of equifinality

A key principle of analysis is that of *equifinality* (Brown & Coles 2011, p.866). The concept of equifinality is associated with early cybernetic, or general systems, thinking (von Bertalanffy 1969). Bateson was also involved in the early cybernetic movement, which aimed to look for generalities across systems that are independent of their components. In the early years of this strand of thinking, it was assumed that systems move from one equilibrium position to another. It was assumed that a system not in equilibrium was in danger of collapse, and would resolve itself as quickly as possible into an equilibrium state, before being triggered out of it by some other event, only to return again to equilibrium. The concept of ‘equifinality’ described how such stability seeking systems would often appear to reach the same equilibrium position from a wide variety of initial conditions.

Later systems thinking, influenced by the mathematics of chaos and complexity, has come to recognise the incredible sensitivity on initial conditions that certain systems display. It has been recognised how systems are able to exist and operate ‘far from equilibrium’ (Juarrero 2002, p.119). One insight here is that complex systems are finite. Although existing ‘far from equilibrium’ is inherently unstable, systems (such as humans) are able to maintain such instability for a finite time before collapsing.

Surprisingly, perhaps, there is evidence that communication within groups of humans follows patterns more closely resembling simpler, equilibrium seeking systems, than the more complex far from equilibrium and chaotic ones (Coles 2013a, p.12-13). The concept of equifinality is used today in some branches of family therapy to describe how family patterns can become constrained (Stroh Becvar & Becvar 2000).

The implication of equifinality, for the analysis of classroom data, is that the search for pattern is likely to be most productive if it begins with the ‘final’ piece of data collected from any group. In other words, if a sequence of recordings is taken over a period of time from the same group, analysis can begin with the last one taken. On the principle of equifinality, this recording should be the one that contains the most established, and hence easiest to see, patterns. Having identified a pattern, the earlier data can be interrogated in order to trace its emergence. Considering the emergence of pattern allows the researcher to avoid the language of cause and effect. From an enactivist perspective, it is not possible to identify direct causes of behaviour in any simplistic way since the entire history of an organism and environment is relevant to what occurs in any particular moment. However, it is possible to *observe* how patterns emerge and change. With the example of

the use of the word ‘conjecture’ offered in Section 3.2, further analysis of this word use over time showed that as the academic year progressed, the word was used less and less. However, all students (in interview) used the word to describe their work in mathematics and from video recordings of lessons, it was evident that students were making and forming conjectures as a regular part of their classwork (Coles 2013b).

3.4 Micro-analysis

The search for pattern may throw up moments in, say, classroom data as being particularly significant (e.g., from multiple perspectives). These moments can often be worthy of further, more detailed analysis. An enactive analysis of language will be interested in the fine-grain of talk. There are micro-analysis techniques for studying language that have evolved from the field of conversation analysis (see Sacks 1995; Goffman 1975; Schegloff 1989; Rampton 2006; Lefstein 2008) that itself grew out of ethnomethodology. Micro techniques of data analysis involve a prescription on researchers concerning where to place their attention. A key component of these techniques, within linguistic ethnography, is to approach small sections of transcript with a slow and repeated reading, keeping some questions in mind. Rampton (2007b, p.4) uses the following questions when considering each turn of dialogue: “‘Why that now?’ ‘What else might have been done here, but wasn’t?’ ‘Why not?’ ‘What were the options available at just this moment, which was selected, why?’”.

There are some difficulties, from the enactivist position, in asking why something was said, in the danger of (over)interpreting some internal event. The following questions allow a similar dwelling in the detail of each turn of a small section of dialogue, while being more in keeping with the enactivist view of knowledge and cognition:

- what pattern does it follow?
- what pattern does it break?
- what distinction is implied?

The last of these questions needs elaboration. If a teacher says, ‘That’s a really good question’, a distinction is being drawn between this question, which is ‘good’, and previous ones that are, by implication, bad. The question ‘what distinction is implied?’ helps alert the researcher to things *not* said (this question is adopted from Laurinda Brown, who refers to seeing the ‘not’ of what people say). The practice of analysing what is not said supports the researcher in developing reflexive sensitivity to the production of linguistic claims (Rampton 2007a, p.596).

By posing the three questions above, as a transcript is slowly read and re-read, analysis of dialogue can take account of immediate historical patterns - how the past constrains what may happen now and the ways that speakers break such patterns. The three questions place a demand on *how* analysis is carried out (Coles 2009). There is an acknowledgment of the role of the observer and an attempt to give an account of the process by which observations are made.

3.5 Meta-communication

One of the patterns that is likely to be of interest, from an enactive point of view, is in the logical level of communication, i.e., the extent to which messages about the talk that is occurring (metamessages) are made explicit. Teachers and students may, at times, communicate about their communications, i.e., offer meta-comments. These are seen as particularly significant since they can operate to frame whole sequences of dialogue and can either be consistent with, or clash with the

communications that actually take place. The interest in meta-communication is linked to the importance of the observer. A meta-communication indicates that the speaker is, to some extent, observing themselves. Meta-communications offer insight into the speaker's observation of the context within which communication takes place.

Another reason for privileging meta-communication is that there is some evidence meta-communications are significant in terms of establishing patterns of talk in a classroom context. One surprising finding from Laurinda Brown and my collaboration, whilst I was still teaching in school, was Laurinda's awareness that year on year there would be something recognisably the same about my year 7 (age 11-12) classes (see Brown and Coles 2008). In one year Laurinda observed students in my year 7 class referring in writing and conversation to the idea that for mathematicians it is 'okay to make mistakes'; in a subsequent year students often referred to the notion that mathematicians 'try to prove themselves wrong'. These statements from students are metacomments and were linked to metacomments made by me, the teacher, in previous lessons. I would make many metacomments each year – it appeared that one idea would be taken up by students and becoming a guiding principle in how they thought about and did mathematics. The idea changed year on year (despite my making similar kinds of metacomments), what stayed the same was a metapattern that there was *one* predominant guiding idea of what it meant in this classroom to be 'becoming a mathematician'. Here is an example of a pattern that connects (see Section 3.2).

4. Into the classroom

In what follows the five mechanisms will be exemplified, to investigate the use of the proposed methodology. The data collection took place between 2007-9. The video data taken from two teacher's classrooms was recorded using a fixed, unattended camera placed at the back of the classroom, pointing to the teacher's board with as wide a lens angle as possible. Students were given the option of either not being involved in lessons that were recorded, or of sitting out of camera shot. In general the backs of around a dozen students' head are visible on the recordings. I took ten recordings of each teacher over the data collection period (2007-9).

Reviewing video recordings from 2008, there was a pattern apparent in the almost identical words (see below) said by both teachers:

TA: can anyone else explain what she's done there
TB: can anybody say what S1 has just said

These extracts were from lessons in 2008, to classes of 11-12 or 12-13 year olds. I saw myself, as head of mathematics at that time, as responsible for developing an expectation of collaboration amongst mathematics staff, and a mutual focus on sharing effective teaching strategies within rich mathematical activities. At the time, I read these almost identical words as evidence of shared practice.

I offer a snapshot of context in introducing each section, to help orient the reading. Following each transcript I then offer my analysis, focusing first on the mechanisms of: search for pattern, micro-analysis, metacommunication, and then illustrating aspects of recursive inquiry and commenting on what can be said about learning.

4.1 Transcript 1 - Teacher A

The transcript of classroom dialogue starting from the quote above is reproduced here. Just before line 1 a student (S4) had offered a proof of a conjecture the class had been working on for several lessons. (Students were numbered according to the order they spoke, we pick up the thread part way through a discussion.) The task was to take a 3-digit number, multiply by 7, then by 11, then by 13 and see what happened (a number such as 762, after the three multiplications becomes 762,762). The proof was trying to capture why this repeating digit pattern must occur and was linked to the realisation that $7 \times 11 \times 13 = 1001$.

Transcript notations: (.) indicates brief pause; (1) 1 second pause; () undecipherable speech; (text) indicates best guess at transcription; // overlapping speech; ? indicates rising intonation; [text] indicates transcriber comment; **bold** indicates stress

- 1 T: oh amazing (.) can anyone else explain what she's done there? (4)
can anyone else explain what she's done there? (.) S1
- 2 S1: ()
- 3 T: (will you) explain this first?
- 4 S1: yeah ()
- 5 T: okay (.) try and do that and then somebody else (.) I want you **all** to listen to this
- 6 S1: what she's done (is that she's timesed)
- 7 T: go on (.) go on in your own words
- 8 S1: a b c is any number a b c if you times that by one thousand its going to be a b c
with three zeros on it /T: lovely yeah/ and you added another a b c
- 9 T: if you added another a b c and you get a b c a b c (.) S2
- 10 S2: it's pretty simple
- 11 T: it **is** pretty simple
- 12 S2: it's basically one two three times one thousand and one
(equals) one two three one two three
- 13 T: does it matter what the one two three is (.) if it was five six seven or five eight nine
- 14 S2: yeah like you said (.) it could be **any** three digit number you times it by a thousand
- 15 T: and then times it by one
- 16 S2: yeah and then times it by one
- 17 T: so this is quite key isn't it (.) the fact that it's a thousand and one (.)
we talked in other lessons whether multiplication was commutative or not
and whether it mattered what order we did it in (.)
so would that be the same as doing it another order (2)
I think we convinced ourselves that multiplying was commutative (.) S3
- 18 S3: () four digits ()
- 19 T: [some teacher talk about use of books skipped here]
what I **like** about S4's proof is its simplicity (2)
and proofs don't have to be complex (.)
so for me I feel convinced by this (.)
S4 has convinced me that when she takes any three digit numbers
and timeses it by seven eleven thirteen she will get a b c a b c
- 20 S3: I've got a theorem as well

The first step of analysis is a further articulation of patterns linked to the statement 'can anyone else explain what she's done there' that led to the research focus on this transcript. The contribution before the transcript begins is the only student contribution in this lesson to which Teacher A asks for another student's explanation. Teacher A comments (line 1) 'oh amazing', in relation to S1s contribution. Line 1 is then followed by three contributions from Teacher A (lines 3, 5, 7) all

supporting students to articulate their ideas. There is an intriguing symmetry of the teacher repeating a student's words (line 11) and a student repeating the teacher's words (line 16). In lines 9 and 15 the teacher completes a student's utterance.

Following the identification of pattern, a second step is consideration of metacommunication. The teacher's actions slow down the discussion at this point in the lesson, dwelling in a student's proof, and leading to Teacher A (meta-) commenting (line 19) about this piece of mathematical behaviour to the whole class. In an earlier meta-comment in this lesson, Teacher A had been explicit to the class that she wanted students to be developing ideas of proof. In the language that TA had established in this classroom, a proof is what is needed to turn a conjecture into a theorem. Meaning is interpreted as the co-ordination of actions in the classroom, for example in the apparently smooth flow of conversation. The role of language in co-ordinating these meanings can be seen particularly in the shared vocabulary of proof and theorem. TA, unsurprisingly, has a significant role on this co-ordination; her talk of proof is followed by a student wanting to describe a theorem (line 20). Students did not come to the school talking about conjectures and theorems (I verified this in interviews). The framework of proof was new to them and in line 20 we get an example of a student taking up TA's language and making a distinction (she has a 'theorem') that, if not new this lesson, is new since TA has taught her. The making of new distinctions is evidence of learning.

In keeping with the enactive interest in meta-communication and TA's expressed intention to work on proof in the lesson, line 19 bears further scrutiny. The three micro-analysis questions (what pattern does it follow? what pattern does it break? what distinction is implicit?) were used to analyse the first three phrases of line 19.

The use of the word 'simplicity' picks up a student comment in line 10 'it's pretty simple'. An implicit distinction in the first phrase is that other proofs are complex, or a perception that proofs are complex. In this class there had been a number of proofs discussed and worked on as a group, some of them challenging. The implied contrast is spelled out in the second phrase of turn 19. The use of 'like' (in the first phrase of line 19) follows the pattern of TA's positive affirmations (lines 1 and 8) of student contributions. The pattern broken in this first phrase, is that up to this point TA's responses have followed up directly the content of the preceding student comment. S3 (at line 18) had made a comment about what happens with four digit numbers. TA does not immediately or explicitly refer to this idea. Instead, she talks about not having much time left in the lesson, needing to do things with books and then returns to talk *about* S4's proof. The communication, in line 19, about the dialogue that has just taken place is an example of explicit metacommunication.

4.2 Transcript 2 - Teacher B (15/9/08)

The transcript from Teacher B's lesson is reproduced below. Just before line 1, S1 had offered an idea. After a pause of a few seconds, another student (S2) made a guttural noise (Huh?), which I took to indicate confusion. Teacher B had been explicit to the class in this lesson about the need for them to listen to and comment on each other's ideas rather than contribute only their own new points. The class were doing the task '1089'. Students choose a 'starting number', apply a process and get a 'finishing number'. (The process is to reverse the number and subtract, then reverse the answer and add.) When you take a three-digit start, the finishing number is generally 1089. Students in this lesson had gone on to explore four digit starting numbers (where there are at least three possible finishing numbers). The board looked something like Figure 2 (re-created from the transcript as it is unclear on the video), different students wrote the numbers to share their findings collectively, underlining the finishing ones with the linked 'start' numbers underneath.

<u>9999</u>	<u>10989</u>	<u>1818</u>	<u>10890</u>
4451	5000	2111	- 12 5210
	4000		3740

Figure 2: A section of the white board

- 1 T: okay S2 is a bit confused
can anybody say what S1 has just said (5)
S1 can you go through it again
- 2 S1: (if there are) all the starting numbers up there have got a nine in on the
- 3 T: these are finishing numbers (.) these are starting numbers
[TB points to two different lists on the board]
- 4 S1: yeah (.) er finishing
- 5 T: all the finishing numbers
- 6 S3: not one eight one eight
- 7 S1: er (.) don't know about that one (.) when you put (.)
if you switch them round the nines'll be together
so it'll be like nine and nine will make eighteen so you've got eight
and then you add the one onto () so the nines go together
- 8 T: okay (.) so why is that important (.) how does **that?** help us

Again, the first step of analysis in an articulation of pattern related to the statement: 'can anybody say what S1 has just said'. Just before line 1, S1's contribution is greeted with an expression of incomprehension (from another student) – the only time this happens in the discussion. In line 1 Teacher B responds to S2 and uses the strategy of getting someone else to explain an idea. Most other student comments in this lesson's class discussions are responded to by Teacher B saying: 'Has anyone got a comment about what S has said?'. Even with the luxury of being able to hear the original idea of S1 over and over again I have found it hard to decipher, it is perhaps not a surprise that no other student offers an explanation in their own words (after line 1), despite a 5 second pause. S1 explains again (line 2), and there is an exchange similar to TA's lesson, in which S1 is supported in articulating his idea (TB suggest a clarification in line 3 and repeats S1's words in line 5). S1 appears to state that all the finishing numbers have a nine in them (lines 2 to 5).

Moving on to consider meta-communication, in line 8 TB asks 'how does that help us' directed at S1, with rising intonation and emphasis on 'that' and discussion moves on to another student. There appears to have been some lack of co-ordination of action in this transcript, in the inconsistency around labels for 'starting' and 'finishing' numbers. TB gestures to two lists of numbers on the board, S1 labels them initially in one way and then swaps, after TB highlights the distinction, in line 3. We can therefore see some confusion of meaning in the transcript. Line 8 can be interpreted as a communication about the preceding dialogue, i.e., as a metacommunication.

In an earlier meta-comment during this lesson, Teacher B had stated an aim in this lesson of getting students responding to each other in discussion. Lines 6 and 7 are the first time in the lesson when the Teacher-Student-Teacher pattern of talk is disrupted, students are commenting to each other, which is something TB had wanted. A more micro-analysis was undertaken on lines 6 and 7.

At line 6, S3 responds to S1's idea (described in lines 2 and 4) with the comment: 'not one eight one eight'. The number S3 refers to (1818) was one of the 'finishing numbers' written on the board

at the time and S3 seems to be drawing a distinction. The finishing numbers on the board were: 10890, 9999, 10989, 1818. S3 appears to be offering a counter-example to S1's idea that all the finishing numbers have a nine in them (i.e., 1818 is a finishing number and it has no nine in it). S1 responds by saying (line 7), 'er, don't know about that one'. The implied distinction here is that S1 *does* know about the 'other' ones. Looking again at the finishing numbers on the board, S1 appears to be acknowledging that his conjecture does not work for the finishing number 1818, but that it does work for the others. S1's turn continues and he refers to switching the numbers around – this was part of the process done to starting numbers in order to get finishing numbers.

In terms of evidence of learning, in line 6 a student makes a distinction between numbers on the board that fit S1's idea and one number that does not and in line 7, S1 appears to accept the distinction. These are sophisticated awarenesses and highly relevant to ideas of mathematical conjecture and proof. There is perhaps a missed opportunity here for TB to highlight and name an awareness with a meta-comment (e.g., 'what S3 has offered there is what mathematicians call a counter-example, because ...'). Part of the enactivist view of learning is that naming a new distinction can be significant in terms of making the distinction available in future.

4.3 Further analysis

An aspect of enactivist methodology highlighted in section 3, was the interaction between data collection and data analysis, and the importance of enriching textual analysis with some process that allows on-going orientation to the context and history of communication. Having noticed the similarity and differences in the two transcripts above and to support my own analysis, I ran a meeting at which I gave out copies of Transcript 1 and 2 and asked for comments and reflections. TA and TB were present, as well as three other mathematics teachers. At this meeting, a distinction was noticed. For TA, the strategy (of getting a student to explain another student's idea) was used at a point when the teacher had made sense of a student comment, and wanted more people in the class to engage with it. For TB, the same strategy was used at a point when she had *not* made sense of what a student said, and when another student had also expressed incomprehension.

In the transcript of TA the pattern of dialogue of the lesson was disrupted in that longer time was spent focused on the point made by a student preceding the use of the strategy, than on any other point in the lesson. TA's use of the strategy served her purposes of slowing down and opening up the discussion at a key moment, following which TA rounded off the whole class discussion and got students to begin working individually or in pairs. In the transcript of TB, following her use of the strategy, a student responds directly to another student (the only instance of this across both transcripts). TB reflected that she had not heard, at the time, what she now saw as a significant student comment (line 6). TB would have liked to have responded, at line 8, by getting S1 and S3 to explain in more detail what they thought they were discussing, if only to highlight to the rest of the class what it means to find a counter-example.

The teacher discussion of Transcript 1 and 2 suggested a distinction in the intentions of TA and TB in their use of a similar strategy – I take this as an aspect of context. However, enactivist analysis of language is committed to staying with what is observable, including the reading of context, so to give credence to the suggestion from the teacher meeting, it should be possible to see this difference (in context) in the text. Returning to Transcripts 1 and 2, with a new distinction in mind, a new view is possible.

In Transcript 1, TA (line 3) suggests where the new student explanation should begin. In line 5, she suggests that she might get someone else to engage in the explanation and then comments to the

whole class that she wants them to listen. TA's comments here are in relation to the others in the class – suggesting a good place to begin an explanation to the others and encouraging those others to pay attention. In contrast, in Transcript 2, TB (line 3) makes a distinction between starting and finishing numbers that it appears S1 may have mixed up in his explanation. Then in line 5, TB repeats part of S1's comment from line 2, but replacing 'starting numbers' with 'finishing numbers'. TB's comments are in relation to what S1 is saying and the attempt to clarify the explanation. It is also relevant that the metacommunication in the transcripts differs and matches the interpretation of the different intentions. TA meta-comments about proof and TB meta-comments about S1's idea not helping the class.

It is, therefore, possible to read the distinction (between contexts) drawn in the teacher meeting in the intentions of TA and TB, in differences between the first few comments of the transcripts and the metacommunications. TA has a suggestion of where the explanation could begin, emphasises that what is coming is important to the class and meta-comments about proof. TB is supporting S1 in stating their idea and meta-comments about the idea not helping. In other words, it can be seen in the transcript that TA has a sense of the explanation that is to come and is already convinced of its importance, emphasising this importance to the rest of the class, in contrast, TB is trying to make sense of the explanation herself.

The idea of using different teaching strategies dependent on whether you have or have not made sense of what a student says, was a new distinction (for me, certainly) that is evidence of teacher learning. One year later, at another staff meeting, TB spoke of having altered her practice in light of this distinction.

6 Conclusion

This article lays out five principles for the study of language in mathematics classrooms, drawn from the enactivist stance in which meaning is taken to be the co-ordination of action, and language is the co-ordination of co-ordination of action, or the co-ordination of meaning. These principles are: to plan a recursive structure to inquiry into language; to begin analysis with a systematic search for pattern; to use patterns identified in the newest data collected, as a lighting on earlier data; to adopt micro-analysis techniques on small sections of transcript; to pay attention to the meta-messages and meta-communications in any dialogue. Taken together, these principles allow something to be said about learning in terms of the making of distinctions.

The principles were investigated via an analysis of two classroom incidents, where near identical words were used with diverging intentions. The different intentions would be hard to detect from direct observation and only came to light through a methodology that took account of the context of the communication by tracking, in a recursive manner, the process in the department of taking videos of lessons, co-watching excerpts or discussing transcripts, and further taking of video recordings. What was said in the classroom could not be interpreted without awareness of the wider context – and this wider context could not be interpreted without detailed study of what was said. The use of a common form of words, even in one department, could not be taken as evidence of common purpose. The analysis reveals a complexity within the classroom that, although available to direct observation, only became apparent using an approach to studying language that took account of the context and history of communication.

Effective study of classroom language needs some way of allowing this history of interaction to inform analysis. The combination of the recursive processes entailed by enactivist research, coupled with more micro techniques for the study of language, provide one way of taking account of context

while focusing on the detail of talk. The use of key questions to keep in mind while analysing transcript data has the effect of forcing attention onto the fine detail of interactions, making it unlikely as a researcher that I will slip into evaluations and habitual ways of thinking. In the same way that Teacher A in the lesson can be seen to slow down discussion at a key moment, the way of working provides a mechanism for slowing down and disrupting researcher interpretation, allowing new distinctions to arise with the potential to see in a richer and more nuanced manner. Metz and Simmt (2015), also taking an enactivist perspective, achieve an integration of context, detail and learning in their analysis of language data through a focus on what it means to be an ‘empathetic second person inquirer’. It is an encouraging signal for the approach sketched above that, in retrospect, these authors were able to identify in their own work the five principles put forward in this article.

In one article it is not possible to explore, in depth, every aspect of a new methodological approach. There is hardly space to critique and consider what is left out. I note, however, that the methodological principles in this article will not guarantee that two different researchers going through the same process will end up making the same distinctions. The aim, rather, is that different researchers would end up agreeing about distinctions and would learn, themselves, through the process, while being able to say something about distinctions made within the transcript. However, micro-analysis is time consuming and can only be done on small sections of talk. The role of pattern in analysis has been left under-theorised and the extent to which the search for the pattern can be ‘unmotivated’. More needs to be said about how the approach allows an analysis of learning. For example, the analysis of the two transcripts points to the importance of *naming* distinctions (a meta-communicative act), in the process of learning. However, is this conclusion simply a result of trying to observe learning through a transcript? In a future article I hope to explore such issues further and also show how the approach sketched above can be applied to larger data sets.

References

- Barwell, R. (2008). Discourse, mathematics and mathematics education. In M. Martin-Jones, A.-M. de Mejia & N. Hornberger (Eds.), *Encyclopedia of language and education, vol. 3: Discourse and education* (2nd ed., pp. 317–328). New York: Springer.
- Bateson, G. (2000). *Steps to an ecology of mind*. Chicago: University of Chicago Press.
- Bateson, G. (2002). *Mind and Nature: A Necessary Unity*. Cresskill, NJ: Hampton Press Inc.
- Blumer, H. (1954). What’s wrong with social theory. *American Sociological Review*, 19(1), 3-10.
- Brown, L., & Coles, A. (2008). *Hearing Silence: Steps to teaching mathematics*. Cambridge: Black Apollo Press.
- Brown, L. & Coles, A. (2011). Developing expertise: how enactivism reframes mathematics teacher development. *ZDM Mathematics Education*, 43(6-7), 861-873.
- Coles, A. (2009). Towards an aesthetics of education. In S. Lerman & B. Davis (Eds.), *Mathematical action & structures of noticing: studies on John Mason's contribution to mathematics education* (pp. 135-146). Rotterdam: Sense Publishers.
- Coles, A. (2013a). *Being alongside: for the teaching and learning of mathematics*. Rotterdam: Sense Publishers.
- Coles, A. (2013b). On metacognition. *For the Learning of Mathematics*, 33(1), 21-26.
- de Freitas, L. (2010). Regulating mathematics classroom discourse: text, context, and intersubjectivity. In M. Walshaw (Ed.), *Unpacking pedagogy: new perspectives for mathematics classrooms* (pp.129-151). Charlotte, NC: Information Age Publishing.
- Edwards, D. (1987). *Discourse and Cognition*. London: Sage Publications Ltd.
- Goffman, E. (1975). *Frame Analysis*. Middlesex, UK: Penguin books.

- Halliday, M. (1999). The notion of “context” in language education. In M. Ghadessy (Ed.), *Text and context in functional linguistics* (pp.1-29). Amsterdam: John Benjamin Publishing Co.
- Herbel-Eisenmann, B., & Wagner, D. (2010). Appraising lexical bundles in mathematics classroom discourse: obligation and choice. *Educational Studies in Mathematics*, 75(1), 43-63.
- Herbel-Eisenmann, B., Wagner, D., & Cortes, V. (2010). Lexical bundle analysis in mathematics classroom discourse: The significance of stance. *Educational Studies in Mathematics*, 75(1), 23-42.
- Juarrero, A. (2002). *Dynamics in action: intentional behaviour as a complex system*. Cambridge, Massachusetts: The MIT Press.
- Khan, S., Francis-Poscente, K., & Davis, B. (2015). Accumulation of experience in a vast number of cases: Enactivism as a fit framework for the study of spatial reasoning in mathematics education. *ZDM Mathematics Education*, 47(2) (this issue)
- Lefstein, A. (2008). Changing classroom practice through the English National Literacy Strategy: a micro-interactional perspective. *American Educational Research Journal*, 45(3), 701-737.
- Maheux, J. & Proulx, J. (2015). Doing mathematics: Analysing data with/in an enactivist-inspired approach. *ZDM Mathematics Education*, 47(2) (this issue)
- Maturana, H., & Varela, F. (1987). *The Tree of Knowledge: The Biological Roots of Human Understanding*. Boston & London: Shambala.
- Maturana, H. (1988). Reality: The search for objectivity or the quest for a compelling argument. *The Irish Journal of Psychology*, 9(1), 25-82.
- Maturana, H., & Verden-Zoller, G. (2008). *The origin of humanness in the biology of love*. Exeter, UK: Imprint Academic.
- Merleau-Ponty, M. (2002). *Phenomenology of perception*. London: Routledge
- Metz, M. & Simmt, E. (2015). Researching mathematical experience from the perspective of an empathic second-person observer. *ZDM Mathematics Education*, 47(2) (this issue)
- Ochs, E. (1979). Transcription as theory. In E. Ochs & B. Schiffelin (Eds.), *Developmental pragmatics* (pp. 43–72). New York: Academic.
- Rampton, B. (2006). *Language in Late Modernity: Interaction in an Urban School*. Cambridge: CUP.
- Rampton, B. (2007a). Neo-Hymesian linguistic ethnography in the United Kingdom. *Journal of Sociolinguistics*, 11(5), 584-607.
- Rampton, B. (2007b). Illustrations of linguistic ethnography in action: a job interview. Resource document. *Ethnography, Language & Communication*.
<http://eprints.ncrm.ac.uk/1481/1/LE%20illustrative%20analyses%20takehome%2012mar073.pdf>. Accessed 15 March 2008.
- Reid, D. (1996). Enactivism as a methodology. In L. Puig & A. Gutierrez (Eds.), *Proceedings of the twentieth annual conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 203-209). Valencia, Spain: PME 20.
- Reid, D., & Mgombelo, J. (2015). Roots and key concepts in enactivist theory and methodology. *ZDM Mathematics Education* 15(2) (this issue)
- Rorty, R. (1989). *Contingency, Irony, and Solidarity*. Cambridge: Cambridge University Press.
- Sacks, H. (1995). *Harvey Sacks Lectures on conversation. Volumes I & II*. Oxford, UK: Blackwell.
- Schegloff, E. (1989). Harvey Sacks - Lectures 1964-1965: An introduction/memoir. *Human Studies*, 12, 185-209.
- Steinbring, H. (2015). Mathematical interaction shaped by communication, epistemological constraints and enactivism. *ZDM Mathematics Education* 15(2) (this issue)
- Stroh Becvar, D., & Becvar, R. (2000). *Family therapy: a systemic integration*. Boston: Allyn & Bacon.
- Thompson, E., & Stapleton, M. (2009). Making sense of sense-making: reflections on enactive and extended mind theories. *Topoi*, 28(1), 23-30.

- Varela, F. (1999). *Ethical know-how: action, wisdom and cognition*. Stanford, California: Stanford University Press.
- Varela, F., Thompson, E., & Rosch, E. (1991). *The Embodied Mind: Cognitive Science and Human Experience*. Massachusetts and London: The MIT Press.
- von Bertalanffy, L. (1969). *General system theory*. New York: George Braziller.
- Wittgenstein, L. (2009). *Philosophical Investigations*. Oxford: Wiley-Blackwell, 4th edition.